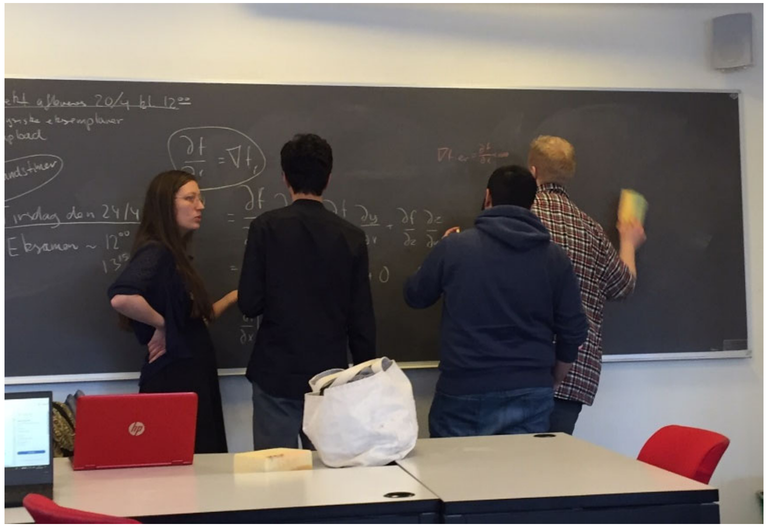
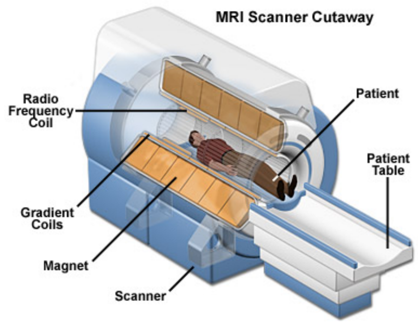
Karsten Schmidt, DTU Compute / LearnT:

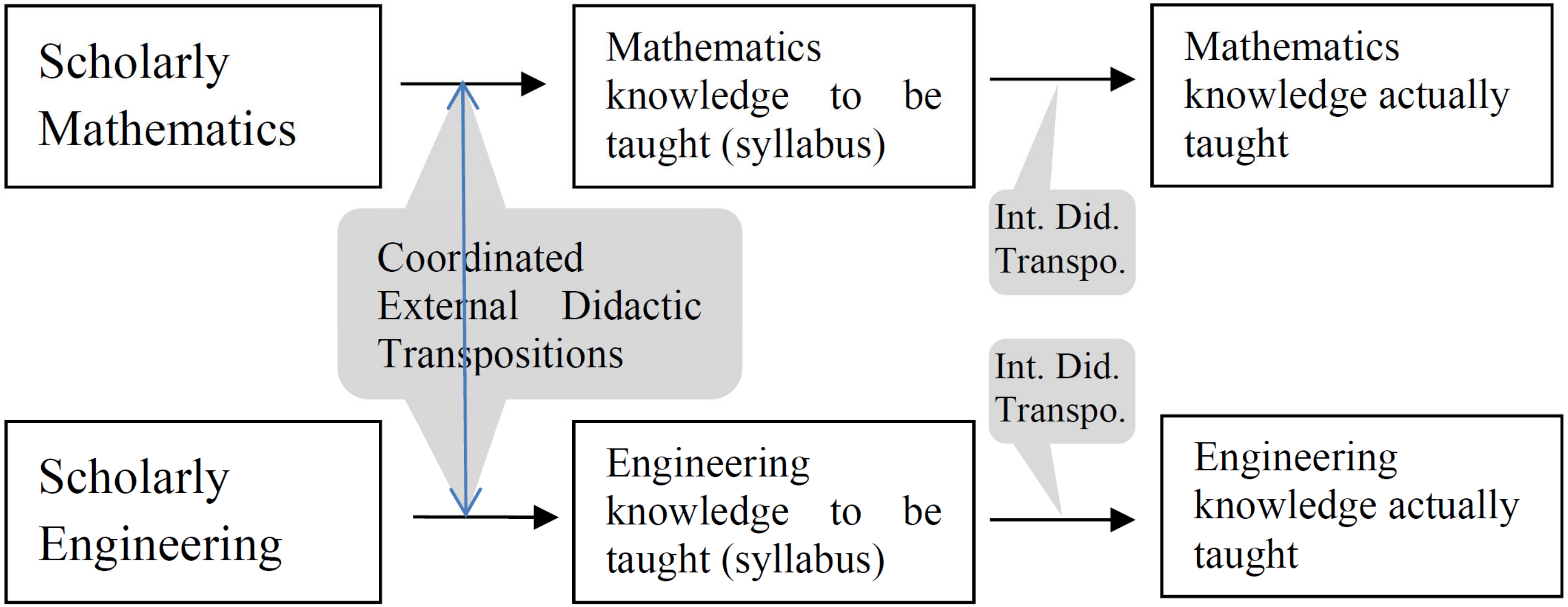
Authentic problems from engineering in introductory mathematics



$$-\nabla^2 \mathbf{A} = \mathbf{Rot}(\mathbf{B}_{\mathrm{rem}})$$



The parallel model for didactic transposition



Karsten Schmidt and Carl Winsløw:
Task design for Engineering Mathematics: process, principles and products, INDRUM 2018

Well known drawbacks of parallel transposition

- Students may experience the mathematics teaching as unmotivated and difficult, which is reflected in relatively high failure rates for some engineering programs
- The knowledge they acquire in the mathematics course may not transfer readily to engineering contexts – that is, students are not able to invest the knowledge acquired in mathematics courses when they need to do so in other courses of the programme

Outline

1. The parallel model for didactic transposition (done)
2. The parallel model and Mathematics 1 at DTU
3. Examples on project work assignments (APEs)
4. How can we analyze the design of the assignments

Mathematics 1 at DTU

”Mathematics is a part of an engineers general education (bildung)”

(DTU’s Bachelor Dean)

One big 20 ECTS math course for all 20 bachelor study programmes!

A:
- Biotechnology
- Chemical Eng
- Environmental Eng
- Human Life Science
- Medicine&Tech
- Quantitative Biology

B:
- Architectural Eng
- Civil Eng
- Earth and Space Phys
- Mathematics&Tech
- Mechanical Eng
- Phys&NanoTech

C:
- Artificial Intelligence
- Design & Innovation
- Electrical Eng
- General Engineeing
- Network Tech
- Software
- Strategic Analysis

How can we motivate the students?

ment (Faroe Islands)

Combining two approaches to teaching

1. The ”ordinary” continuous treatment of the math subjects:
- Lectures
- Group exercises (supported by TAs)
- Homework assignments

2. The project exercises (group work, no lectures):
- Thematic exercises (illustrates math subjects just introduced)
- The ”big” 4-weeks project at the end of second semester (combines several subjects and should give a true glimpse of research)

Necessary conditions:
- Intensive use of CAS (Maple)
- Digital Assessment (MapleTA)

Mathematics 1 – week by week

Autumn:

1. Intro to complex numbers
2. …continues
3. …continues
4. …continues
   Theme 1: Complex functions
5. Linear equations
6. Matrix-algebra
   Theme 2: Networks
7. Vector spaces
8. Linear transformations
   Theme 3: Deformations
9. Function spaces
10. Eigenvalues
11. Linear diff. equations
12. Systems of diff. equations
13. Theme 4: Diff. equations

Spring:

1. Functions of two variables
2. Taylor for several variables
3. Max/Min
   Theme 5: Optimization
4. The Riemann integral
5. Surface integrals
6. Volume integrals
   Theme 6: Integration
7. Vector fields. Flux
8. Gauss's Theorem
9. Big project execise
10. Big project execise
11. Big project execise
12. Project exam
13. Stokes's Theorem
    Theme 7: Forest Fire

The project work assignments

The form:

- 20-30 more or less challinging tasks
- 4 to 29 pages, average 11
- The mathematical model is typical given
- From more closed to more open-ended tasks

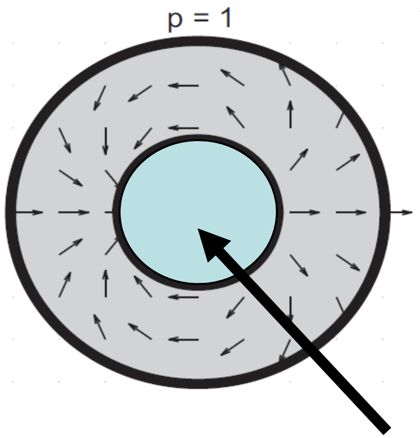
Who are the authors?:

- Mathematicians from DTU Compute
- Colleagues from other DTU departments (two types)
- Collaboration
- PhD students

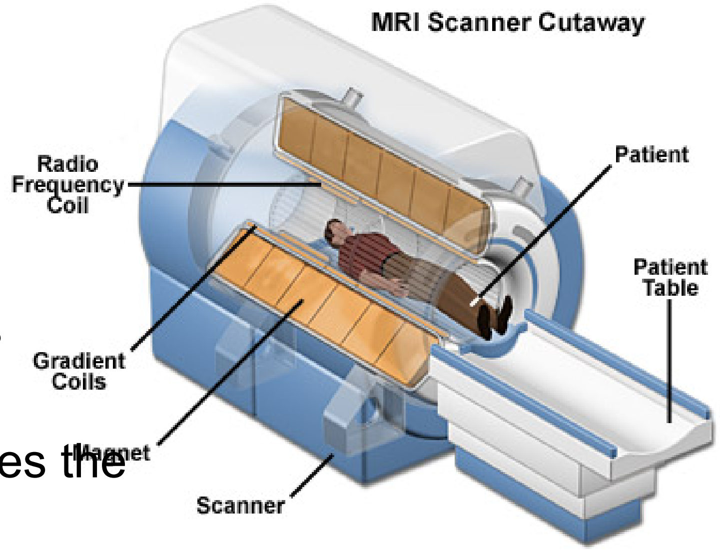
The assignments are very different!

Permanent Halbach Magnets, Spring 2018

Permanent Halbach Magnets



How to create a concentrated homogenious

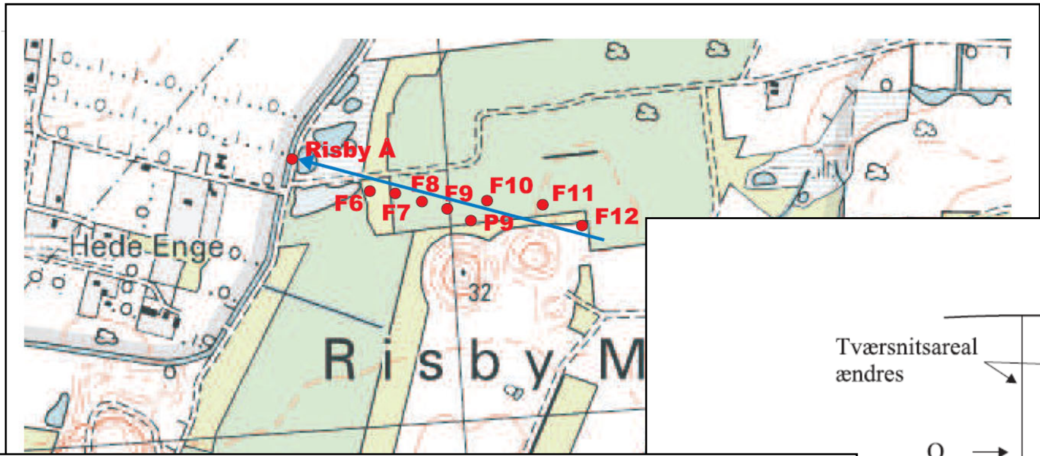


By Maxwell and Stokes the mathematical model:

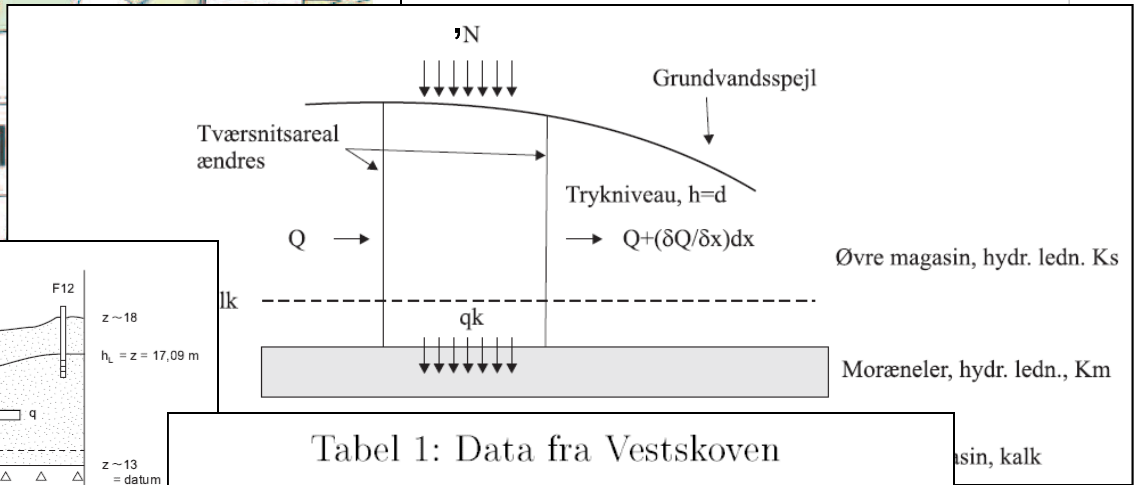
$$-\nabla^2 \mathbf{A} = \mathbf{Rot}(\mathbf{B}_{\text{rem}})$$

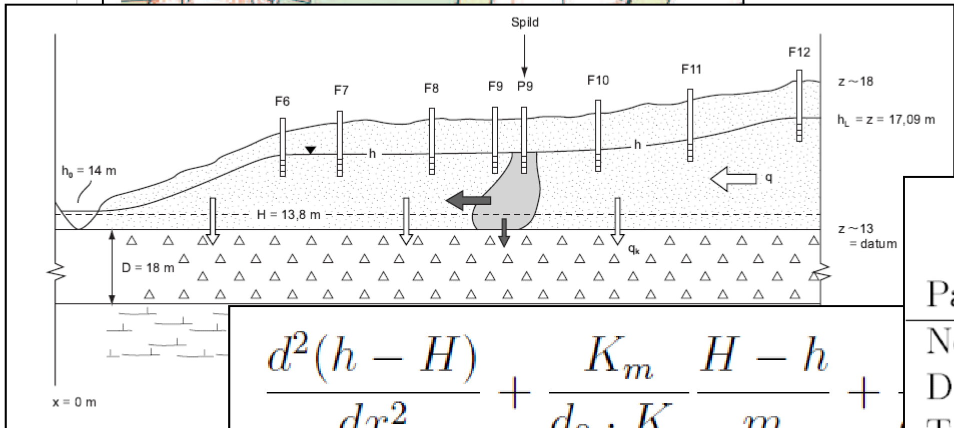
[The text](#) [Paper](#)

Flow of Groundwater in Vestskoven



Author:
Ass. Prof. *Peter Engesgaard*
expert in the field





$$\frac{d^2(h-H)}{dx^2} + \frac{K_m}{d_0 \cdot K_s}\frac{H-h}{m} + \cdots$$

Tabel 1: Data fra Vestskoven

Parameter	Værdi	Enhed
Netto-infiltration, N	0.2	m/år
Datum, z_0	13	m
Tykkelse af moræneler, m	18	m
Trykniveau i kalk, H	13.8	m
Porøsitet i øvre magasin, θ	0.21	-
Længde af tværsnit, L	500	m
Fasthold tryk, h_0	17.09	m
Fasthold tryk (åen), h_L	14.00	m

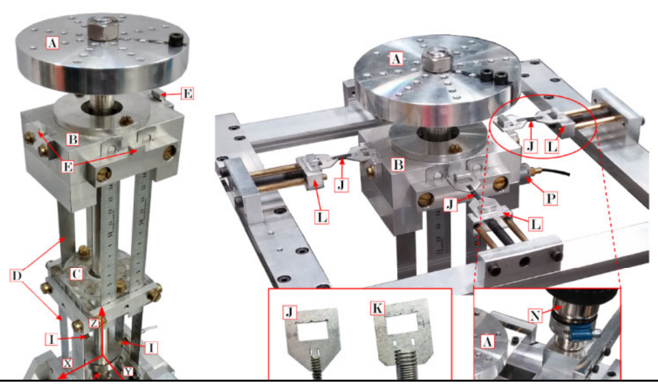
Darcy's law: $Q = -K_s \cdot A \dfrac{\partial h}{\partial x}$

Oscillations in axle bearings systems

Svingninger i aksel-leje-system med integrerede formhukommelsesmetaller

Søren Enemark

Januar 2016



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Referencer

Enemark, S. og I. F. Santos (2016). "Rotor-bearing system integrated with shape memory alloy springs for ensuring adaptable dynamics and damping enhancement – Theory and experiment". *Journal of Sound and Vibration*. DOI: 10.1016/j.jsv.2016.01.023.

Anthrax – attack, escape and rescue

Miltbrand - angreb, flugt, og redning

v./ Irina Borodina, DTU Biosys og Steen Markvorsen, DTU Matematik

Opgavebebeskrivelse

01005 Matematik 1 - FORÅR 2011

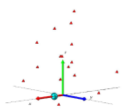
Det er projekt-opgavens primære formål at introducere nogle af de begreber og metoder, der benyttes ved analyse og modellering af særlig farlig forurening, der spredes via diffusion i luften, in casu miltbrand, over store geografiske områder. De simplificerende matematiske og geometriske modellerings-antagelser giver mulighed for strategiske overvejelser f.eks. vedrørende udformning af redningsplaner, valg af redningsruter, placering af mobile behandlings-stationer, den bedste udnyttelse af eksisterende hospitaler, osv. når forureningen er udløst.

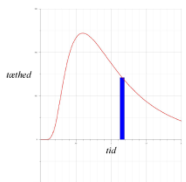
Efter frigivelse (åbning) til tiden $t = 0$ af miltbrand-sporerne i et givet punkt i rummet er tætheden af sporerne givet ved en (rimelig simpel) funktion $p(x, y, z, t)$ af sted og tid, som opfylder *diffusionsligningen*

$$\Delta p = Q \frac{\partial}{\partial t} p \quad ,$$

hvor Δ er Laplace-operatoren i $\mathbb{R}^3$ og Q er en diffusionskonstant, som afhænger af tryk, temperatur og luftens konsistens iøvrigt, samt af sporernes geometri.

Opgaven går ud på at analysere forskellige konsekvenser af disse grundlæggende model-antagelser samt vurdere de nødvendige modifikationer som kan skyldes vind, tyngdekraft, eller eventuel 'refleksion' af sporer fra $(x, y)-$basis-planen (afhængig af om der lokalt er tale om jord, vand, gade, by, eller andet terræn). I inhalerings-højde over basis-planen undersøges enhver af tæthedsfunktionerne p til ethvert tidspunkt som en funktion af de to koordinater x og y - med velkendte metoder. Langs forskellige givne flugtruter inhalerer du forskellige antal sporer. Det totale antal afhænger dels af tæthedsfunktionen p langs den rute du løber, og dels af vejrtrækningsfrekvensen (der jo også afhænger af din fart). En af udfordringerne er så, om du med en detaljeret givet og oplyst spore-tætheds-funktion p kan finde (d)en rute fra ethvert givet start-punkt til et givet andet punkt (en behandlingsstation eller et hospital), sådan at forskellige behandlinger kan sættes i værk - i tide, dvs. således at det totale antal inhalerede sporer og deres allerede påførte skadevirkning (via inkubation og eksponentiel vækst af bakterier) ligger indenfor et af de sikreste af de medicinsk veldefinerede behandlingsvinduer, så du kan nå at blive reddet.





Figur 1: *Animeret online.* Diskret diffusion og måling (med den blå bevægede sensor) af tætheden p langs $x-$aksen som funktion af tiden. Det samlede antal sporer, der inhaleres hvis du løber med sensoren langs den rute, er essentielt integralet af funktionen i figuren til højre.

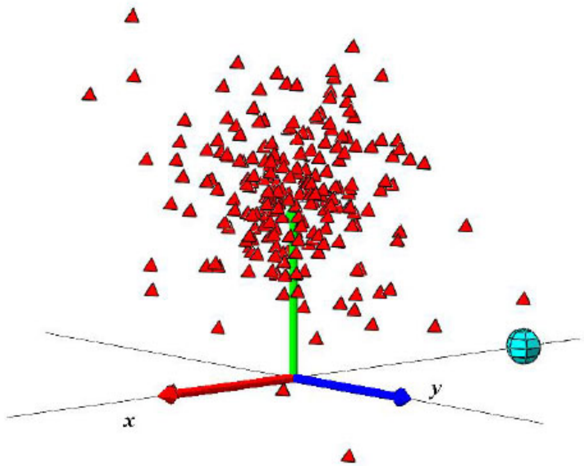
Problemstillingerne behandles blandt andet ved hjælp af Maple. I evalueringen af projektforløbet tages der hensyn til, at projektopgaven kan løses på flere måder, og at man kan lægge sin primære indsats på forskellige delspørgsmål. Det afgørende er, at rapporten dokumenterer, at der er udført seriøst arbejde.

Authors:

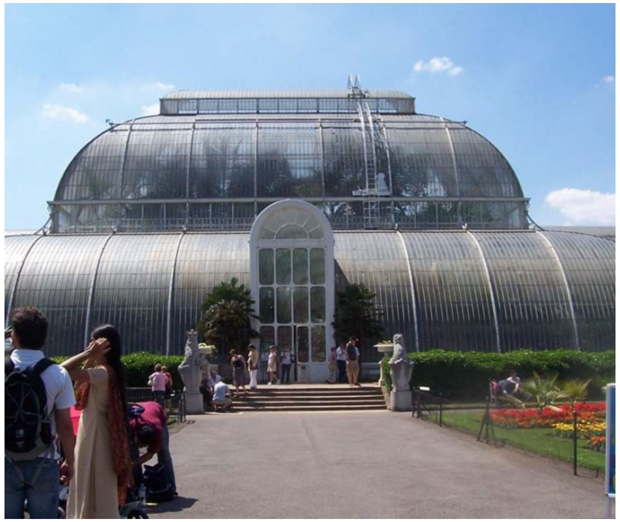
Irina Borodina, DTU Biosys and
Steen Markvorsen, DTU Compute

Involves:

- Diffusion equation $\Delta p = Q \frac{\partial}{\partial t} p$
- Parametrized curves
- Integration in several variables
- Visualizations by Maple



Solar energy absorption in glass houses

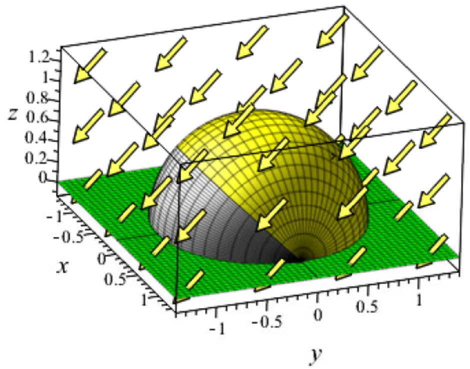


Author:
Karsten Schmidt, DTU Compute

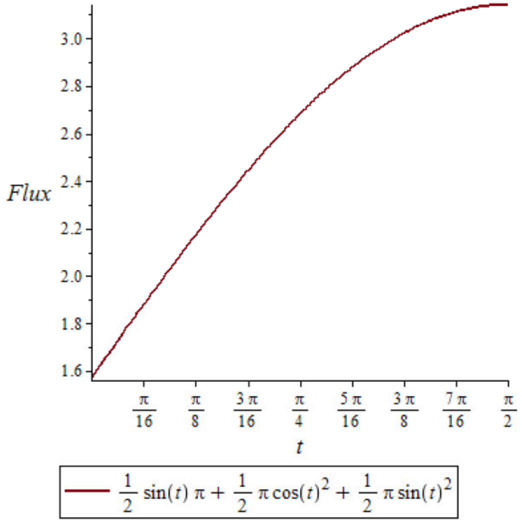
Involves:
- Elementary geometry
- Parametrized surfaces
- Vector fields and flux
- Gauss’s divergence theorem

A group-paper on glass houses, clipses

Analyzing a hemisphere



(a) Sol- og skyggebelagt enhedshalvkugle kl. 9 om formiddagen.



(b) Den positive flux gennem halvkuglen den første halvdel af dagen.

Efter dette tidspunkt skal hele kalottens flux medtages. Med resultaterne af disse overvejelser kan enhedskuglekalottens energioptag gennem en hel dag nu beregnes ved følgende:

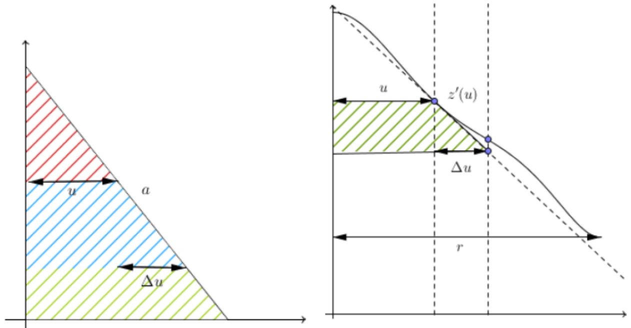
$$E_1 = \int_0^{t_{oplyst}} \left[\int_{-\sqrt{3}/2}^{sk\ae ring_1(t)} \int_{-\sqrt{3/4-u^2}}^{\sqrt{3/4-u^2}} \mathbf{N}(u,v)\cdot\mathbf{V}(t)\,dv\,du \right] dt \approx 0.0829 \qquad (5.16)$$

$$E_2 = \int_0^{t_{oplyst}} \left[\int_{sk\ae ring_1(t)}^{sk\ae ring_2(t)} \int_{v_{skyggegr\ae nse}(u,t)}^{\sqrt{3/4-u^2}} \mathbf{N}(u,v)\cdot\mathbf{V}(t)\,dv\,du \right] dt \approx 1.2188 \qquad (5.17)$$

$$E_3 = \int_0^{t_{oplyst}} \left[\int_{sk\ae ring_2(t)}^{\sqrt{3}/2} \int_{-\sqrt{3/4-u^2}}^{\sqrt{3/4-u^2}} \mathbf{N}(u,v)\cdot\mathbf{V}(t)\,dv\,du \right] dt \approx 0.0829 \qquad (5.18)$$

A group-paper on glass houses, clipses

Surfaces of revolution:



(a) Konceptionel 2D tegning af et vandret kegleudsnit (blåt).

(b) Den givne ledekurve inddeles i stykker med bredden Δu, der approksimeres med et vandret kegleudsnit.

Figur 11.3: Figurerne anvendt i forbindelse med udledningen af integralfo

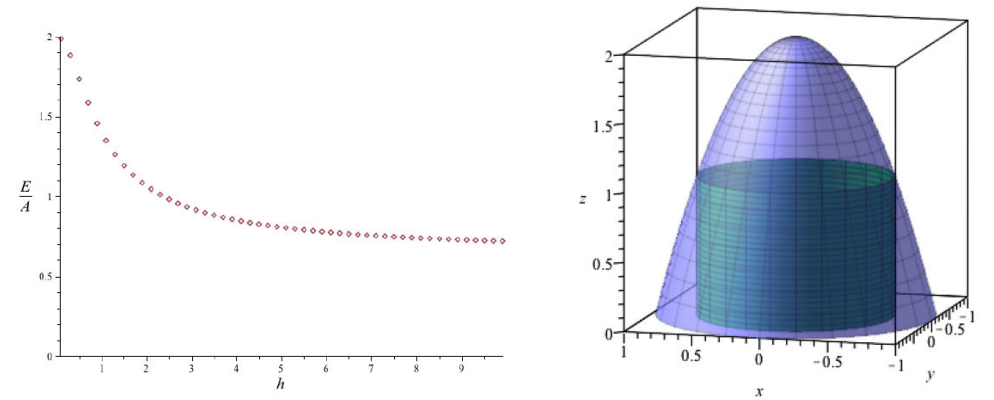
$$E_{tot} = \int_0^1 \int_{-\pi}^0 \int_{t_{kritisk,0}}^{\pi} \mathbf{V}(\mathbf{r}) \cdot \mathbf{N} \, dt \, dv \, du + \int_0^1 \int_0^{\pi} \int_0^{t_{kritisk,1}} \mathbf{V}(\mathbf{r}) \cdot \mathbf{N} \, dt \, dv \, du$$
$$= r^2\pi + 2r^2 \,\text{EllipticE}\left(\sqrt{-\frac{h^2}{r^2}}\right) \quad (11.5)$$

Dette udtryk kan simplificeres ved at faktorisere og anvende at $\sqrt{-1} = i$. Dette gøres herunder, mens der indføres størrelsen $a = \frac{h}{r}$.

$$E_{tot} = r^2\pi + 2r^2 \,\text{EllipticE}\left(\sqrt{-\frac{h^2}{r^2}}\right) \quad (11.6)$$

(11.7)

Optimization under constraints:



(a) Energioptag pr. arealenhed som funktion af højden for en nedadvendt omdrejningsparaboloide. Bemærk at E/A bliver større, jo lavere paraboloiden er, idet der vil være mindre skygge på en lav paraboloide.

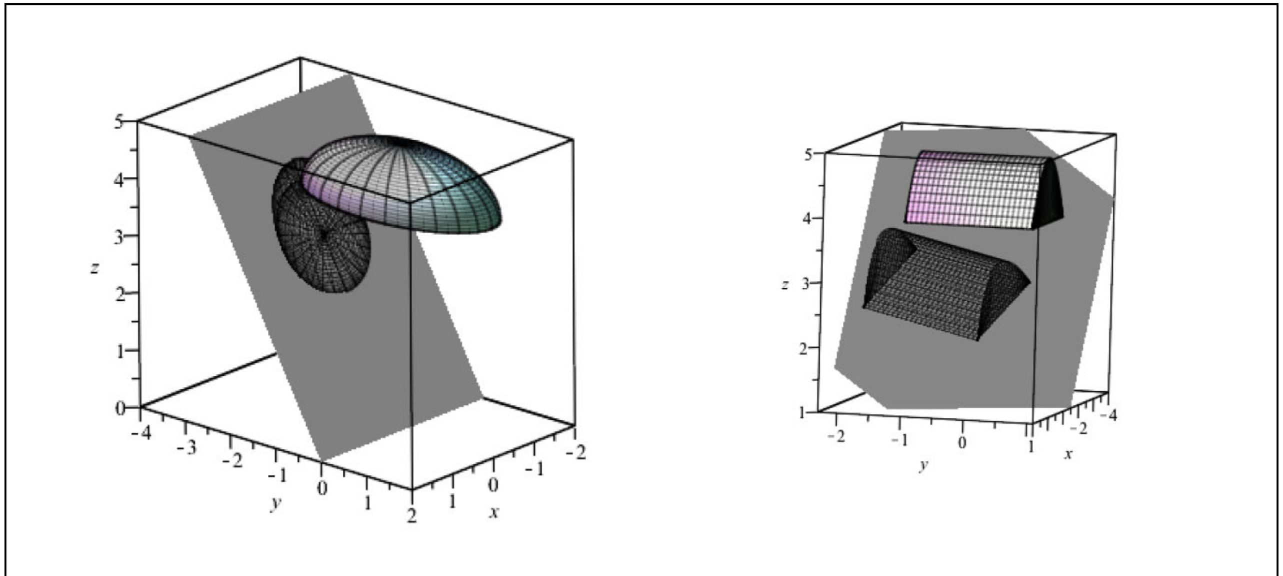
(b) Nedadvendt omdrejningsparaboloide indeholdende en cylinderflade med konstant volumen. Begge flader er optimeret, for at give maksimalt energioptag pr. arealenhed

Figur 13.1: Figurer anvendt til modellering af nedadvendt omdrejningsparaboloide med indre cylinder.

A group-paper on glass houses, clipses

DTU

Using Gauss's Divergence Theorem:



(a) Projektion af punktmængden A på planet α_t til tiden $t = \pi/6$.

(b) Projektion af solfangerhus drejet vinklen $s = \pi/3$ på planet α_t til tiden $t = \pi/5$.

$$B_{\mathcal{F}}^{+} = \frac{1}{2}\,\mathrm{Flux}_{\mathrm{projektionen}} = \frac{1}{2}\int_{\mathcal{F}} \mathbf{V}\cdot\mathbf{n}\,\mathrm{d}\mu = \frac{1}{2}\int_c^d\int_a^b \mathbf{V}\cdot\mathbf{n}\cdot|\mathbf{N}|\,\mathrm{d}u\,\mathrm{d}v = \frac{1}{2}\int_c^d\int_a^b |\mathbf{N}|\,\mathrm{d}u\,\mathrm{d}v$$
$$= \frac{1}{2}\,A_{\mathrm{projektionen}} = A_{\mathrm{skyggen}} \qquad (14.2)$$

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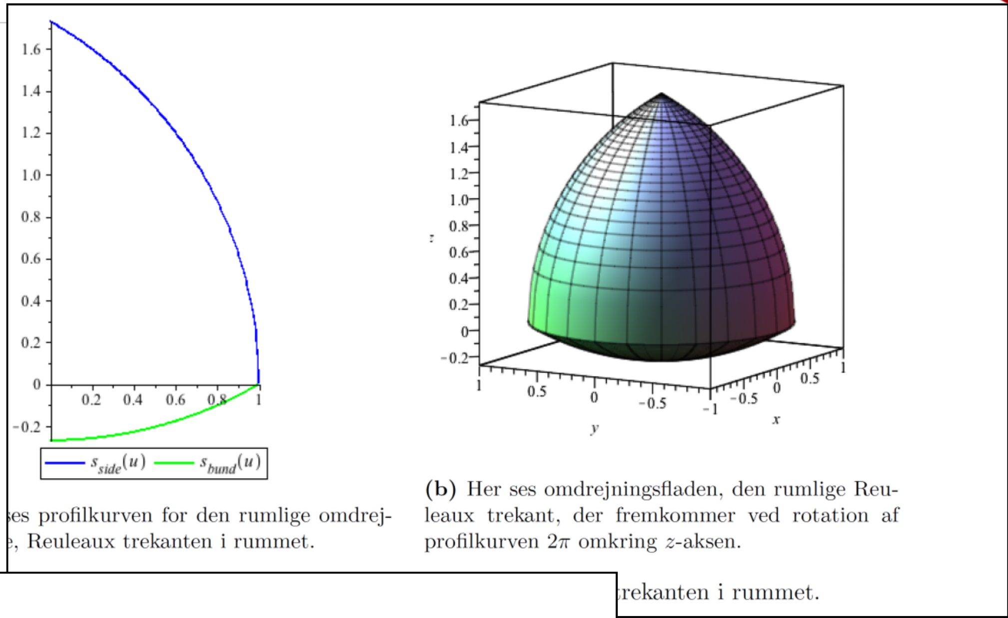
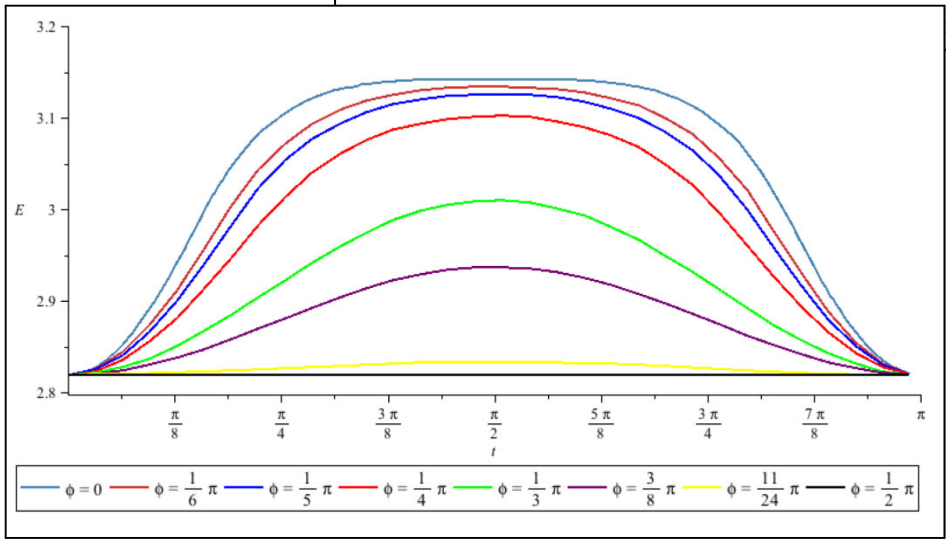
Finding the optimal angle for Copenhagen





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The list of content:

Indhold

The report:	42 p
Maple closure:	119 p
In total:	161 p

Research Questions

Karsten Schmidt and Carl Winsløw:
Task design for Engineering Mathematics: process, principles and products, INDRUM 2018

RQ1. How could the identification and transposition of APEs be organised, given the academic and institutional separation of university mathematics teachers from their colleagues in engineering?

RQ2. What *didactic variables* are relevant to the construction of assignments based on APEs?

10 didactic variables, 1-4

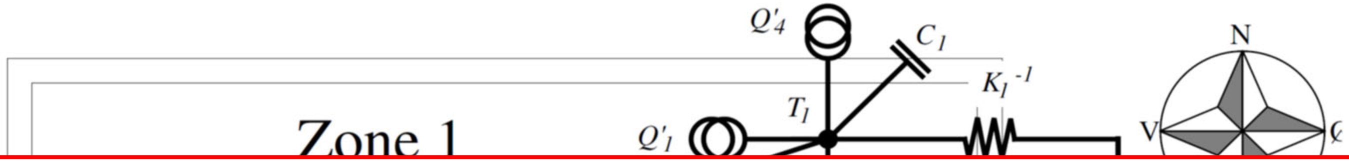
Didactic variable (DV):	**Aim of designers:**
DV1 What breadth of content areas from Mat1 are needed to solve the assignment? What depth of use?	As many as possible, preferably involving new combinations. Depth beyond “standard tasks” required.
DV2 What new mathematical contents are introduced?	Contents in continuation of Mat1, not excessive for students to cope with
DV3 How must/can *Maple* be used? - DV3a How *essential* is the Maple use? - DV3b What types of Maple functions (numerical, symbolic, graphical…) are relevant? - DV3c Are the relevant use known or new to students? - DV3d Is there *black box* use of *Maple* ? - DV3e What parts of the *Maple* use are prescribed?	*Maple* should mostly be used to: avoid tedious computations, and for tasks which the students could not handle otherwise
DV4. What is the ”theme” and source of the problem the project attacks	Origin in APE, if possible source in paper or ongoing research in engineering

10 didactic variables, 5-10

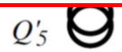
Didactic variable (DV):	Aim of designers:
DV5. Breadth of engineering problem – are more disciplines involved?	Ideally more than one branch of engineering involved
DV6. How is the mathematical model established and worked with?	Ok if model is given in the assignment, but the students should work with its details and structure
DV7. How realistic is the model?	As much as possible for the students
DV8. How are data used?	Data from the source, used as there
DV9. Should the students look up information outside assignment?	This is not a main aim, except students should use Mat1 course material
DV10. How complete answers does the model give to the main problem?	Clear and definite answers/points, to give students a satisfying experience

Presentation of one APE-sample

Heat flow in a house – simulation and dimensioning



The building sector accounts for about 40% of the total energy consumption in Denmark. It is a common assumption that there is a large unrealized potential for reducing the consumption (…) in a financially sound way. This requires knowledge of the physical processes which affect the energy consumption of buildings, the financial aspects of the construction and maintenance of buildings, as well as the mathematical methods used to compute these.



Model for a simple one-room-house:

$$C\frac{dT_i}{dt} = K(T_e - T_i) + P(T_{set} - T_i)$$

Presentation of one APE-sample

Heat flow in a house – simulation and dimensioning

Didactic Variable	Points
1. Breadth of Mat1 content	2
2. New math content	0
3. Use of Maple	1
4. Source of the problem	2
5. Breadth of Eng. problem	2
6. Introduction of the math model	1
7. Realism of the math model	2
8. Use of data	2
9. Need to use external resources WWW)	0
10. Satisfying answers	1

Analysis of current Mat1 projects 1/3

Project name (shortened in a few cases)	1	2	3	4	5	6	7	8	9	10
Oscillations in Axle-bearing Systems	2	0	1	2	1	1	2	2	1	2
Micro/Nano Cantilever Based Mass Sensor	1	2	1	2	1	1	2	1	1	1
Enzymatic Hydrolysis of Cellulose	1	1	1	0	1	2	1	1	0	1
Modelling 2D Halbach permanent Magnets	1	2	2	2	1	2	2	1	0	2
Factorization of Integers	0	2	2	0	1	0	1	0	0	1
Heat flow in house – simulation, dimensioning	2	1	2	2	2	1	2	2	0	1
Quantum Mechanics in a Nutshell	2	0	1	2	2	1	2	2	0	1
Red Blood Cells – Optimization in Nature	1	2	2	1	1	2	1	0	0	2
Utilization of the Waste Product Whey	1	0	1	2	1	2	1	1	0	2
Forced Pendulum	1	2	2	1	1	2	1	0	0	2
Stability in Chilled Tank Reactor	1	2	1	1	1	1	1	1	0	2
Optimization of Work Cycles	2	1	2	2	1	1	1	0	0	2
GPS and Geometry	1	2	2	2	2	1	1	1	2	1

Analysis of current Mat1 projects 2/3

Oscillations in Grid Constructions	2	2	1	2	1	2	1	0	1	1
Groundwater Flow in the Forest Vestskoven	1	2	2	2	1	1	2	2	0	1
Internet Hit lists	1	2	2	2	1	1	2	2	1	2
Short Circuit in Electric Networks	1	2	2	1	1	1	1	0	0	2
Simulation of Stretch Reflex	1	1	1	2	1	1	1	0	0	1
Parking Orbits of Satellites	2	1	0	1	1	2	1	0	1	2
Solar Energy Absorption in Curved Glass houses	2	0	2	1	1	1	1	0	0	1
Flow in Chemical Reactors	2	0	1	1	1	2	1	0	1	2
Finite elements in One Dimension	1	2	2	2	1	2	1	0	1	1
Geodesic Curves	1	2	2	0	0	2	2	0	0	2
The Brains Glycose Metabolism	1	2	2	2	1	1	2	1	1	1
Resistors and Markov Chains	1	2	2	1	1	1	1	0	0	2
Dosage of Anaesthesia	2	1	1	2	1	2	1	1	0	2

Analysis of current Mat1 projects 3/3

Project name (shortened in a few cases)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Anthrax – Attack, Escape and Rescue	1	2	2	2	1	2	1	1	2	2
Decomposition of PCE	1	1	1	2	1	1	1	2	1	0
Modelling Concrete Moulding	1	2	2	2	2	1	2	2	2	1
Soap Membranes	1	2	2	1	1	1	2	0	0	1
Distribution of Electrons in Semiconductors	1	2	2	1	1	1	2	0	0	1
Methane Concentration Profiles in Soil	1	2	1	1	1	1	1	1	0	2
Train Running in the Alps	2	1	2	1	2	2	1	0	1	1
Proteins' 3 Dimensional Structure	0	2	2	2	1	1	1	2	1	1
Reaction Kinetics	1	2	2	1	1	1	2	1	0	2
Error Correcting Codes	0	2	2	1	1	1	1	0	0	2
Phononic bandgaps	2	0	2	2	2	2	1	0	0	2

Preliminary conclusions

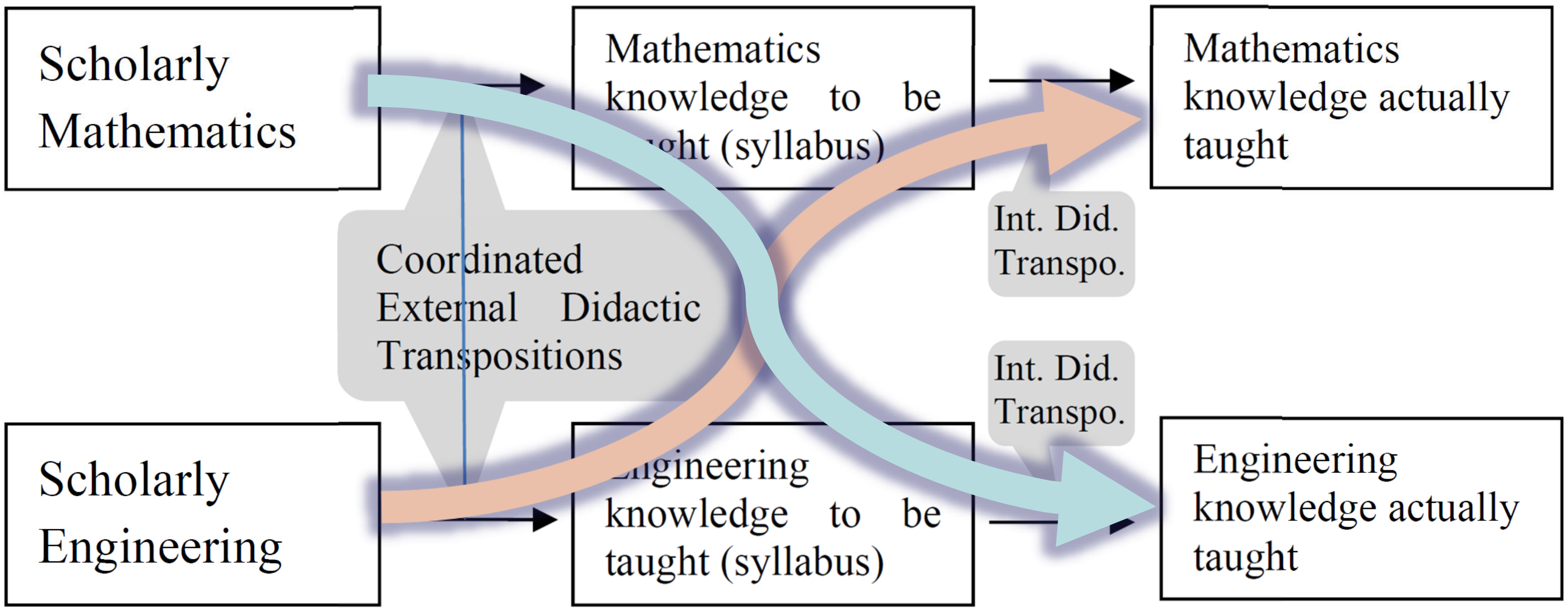
- This is not a measure for the didactic quality!
- Can be an explicit basis for discussion the aim of projects
- Tool for systematizing the design process
- Useful for investigating the effects of the project work

Work in progress:

DTU Fotonik in collaboration with DTU Compute:
Optical fibers: Propagation constant and transverse distribution of electric fields

DTU Compute in collaboration with Greenwood Engineering and others:
Rolling resistance: Reducing the energy consumption due to rolling by modifying the surface texture and evenness of asphalt pavements

The parallel model for didactic transposition



Thank you for attention!