Opgave 1

$$P(z) = P_1(z) \cdot P_2(z)$$

1) $P_1(z) = z^2 + 4z + 4$

$d = 4^2 - 4 \cdot 1 \cdot 4 = 0$, $r = \frac{-4}{2 \cdot 1} = -2$ med am $= 2$.

$P_2(z) = z^2 - 6z + 10$

$d = (-6)^2 - 4 \cdot 1 \cdot 10 = -4$ $\left.\begin{matrix} r_1 \\ r_2 \end{matrix}\right\} = \frac{-(-6) \pm 2i}{2 \cdot 1} = 3 \pm i$

2) $P(z) = (z+2)^2 \cdot (z - (3+i))(z - (3-i))$

De 4 rødder fremgår af faktoriseringen.

3) $$\frac{P(z)}{z+2} = (z+2)(z - (3+i))(z - (3-i))$$
$$= (z+2)(z^2 - 6z + 10)$$
$$= z^3 - 4z^2 - 2z + 20$$

$a = 1$, $b = -4$, $c = -2$, $d = 20$

Opgave 2

1) koefficientmatrix:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a-1 & 1 \\ 1 & 1 & 1-a^2 \end{bmatrix}$$

$$\det(\underline{\underline{A}}) = 1 \cdot ((a-1)(1-a^2) - 1) - 1 \cdot (1 - a^2 - 1) + 1(1 - (a-1))$$
$$= -a^3 + 2a^2 = a^2(2-a)$$

(2)

$\det(\underset{\sim}{A}) = 0 \iff a=0$ eller $a=2$

2). $a=-1$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & 0 & -1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$a=0$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right]$

Ligningsystemet er inkonsistent.

Ingen løsning.

$a=2$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -3 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5/4 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5/4 - t \\ t \\ -1/4 \end{pmatrix} = t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 5/4 \\ 0 \\ -1/4 \end{pmatrix}, \quad t \in \mathbb{R}$$

Opgave 3

1) $_e\underline{\underline{M}}_v = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

Det ses klart at de to søjler er lineært uafh og dermed udspænder de 2 søjler R^2. $(\rho(_e\underline{\underline{M}}_v)=2)$.

2) $_e\underline{\underline{F}}_e \cdot \underline{v}_1 = \begin{bmatrix} 6 & 4 \\ -8 & -6 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\underline{\lambda_1 = 2}$

$_e\underline{\underline{F}}_e \cdot \underline{v}_2 = \begin{bmatrix} 6 & 4 \\ -8 & -6 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\underline{\lambda_2 = -2}$

3) $_v\underline{\underline{F}}_v = {_v\underline{\underline{M}}_e}\, {_e\underline{\underline{F}}_e}\, {_e\underline{\underline{M}}_v} = {_e\underline{\underline{M}}_v^{-1}} \cdot {_e\underline{\underline{F}}_e}\, {_e\underline{\underline{M}}_v}$

$= \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ Ovenstående kan bruges som kontrol, afbildnings matricen opskrives direkte ved kendskab til egenværdier.

4) $_vF(\underline{u}) = {_v\underline{\underline{F}}_v} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

$F(\underline{u}) = 2\underline{v}_1 - 6\underline{v}_2$

Opgave 4

1) Egenværdier og egenvektorer for $\underline{\underline{A}}$:

$$\lambda_1 = 1, \quad \underline{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \lambda_2 = -1, \quad \underline{v}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 \underline{v}_1 e^{\lambda_1 t} + c_2 \underline{v}_2 \cdot e^{\lambda_2 t}, \quad c_1, c_2 \in \mathbb{R}$$

$$= \underline{\underline{c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-t}}}, \quad c_1, c_2 \in \mathbb{R}$$

2)

$$\begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = -2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + 3 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-t}, \quad t \in \mathbb{R}$$

$$= \underline{\underline{\begin{pmatrix} 2 \\ -2 \end{pmatrix} e^{t} + \begin{pmatrix} -3 \\ 6 \end{pmatrix} e^{-t}}}, \quad t \in \mathbb{R}.$$