Mathematics for Non-mathematicians – Rooms for Improvement

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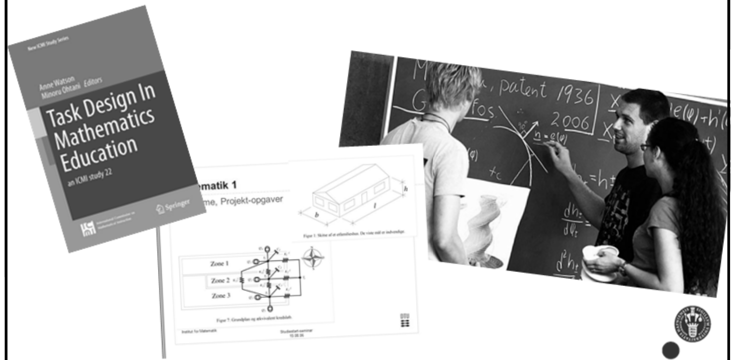


An anecdote to begin with…

My first teaching assignment (1994-1998): *Mathematics for Biology students*

Challenge: first semester course with 250 students who had not chosen and mostly did not like to study mathematics

Basics of 1-var. calculus and differential equations "for later use"

Task inventory used : "end of chapter" exercises from Gulliksen

1995-revision of course: new textbook on "Mathematics for the biosciences" (focus on modelling) and designing new formative and summative tasks; same mathematical contents

New kinds of task (sample):

Consider a single species fishery model ($N = N(t)$):

$$\frac{dN}{dt} = \frac{r}{k}(k - N)N - H(N,t)$$

1. If $H(N)$ is a constant, what is its maximum sustainable value?
2. What kind of fishery does $H(N) = cN$ correspond to?
   What is the maximal sustainable yield? Compare with 1.

TODAY: TWO ROOMS FOR IMPROVEMENT

(1) AT "MICRO LEVEL" (TEACHER'S):
TASK DESIGN

(2) AT "MACRO LEVEL" (CURRICULUM):
MATHEMATICAL NEEDS OF NON-MATHEMATICIANS

(1)

Task Design

"Tasks" (in a large sense) have two main functions in mathematics teaching:

Formative: students can learn mathematics from solving tasks

→ Tasks as a teaching tool

Summative: students can demonstrate their knowledge by solving tasks

→ Tasks as an evaluation tool

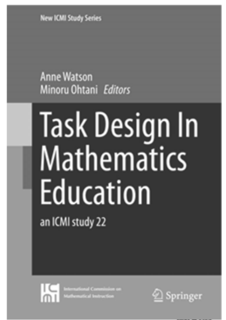
Alignment: connection between above

Problem: not all mathematical tasks can be

- solved in short time, individually
- with short written solutions that are easy to grade "objectively"

Backwash effects from summative to formative.

Recommended reading

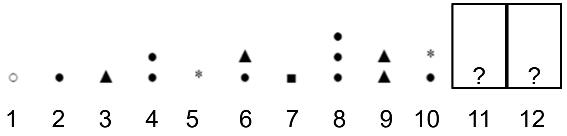


2015

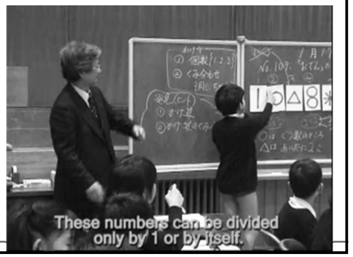
A beautiful example (Tsubota, 2006)

1

Consider the following symbols. Can you find the missing ones?

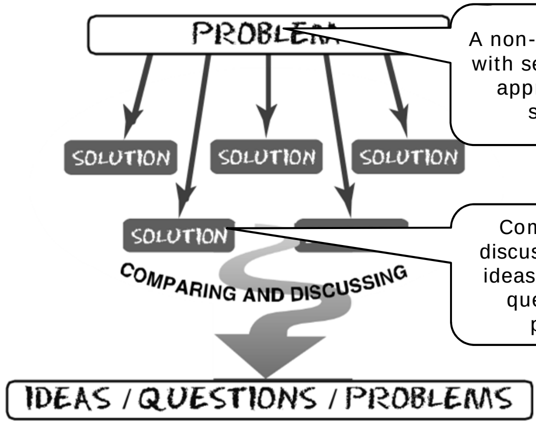


In a grade 4 "open lesson" (Tsukuba-dai fuzoko school):



"Open ended approach" (Nohda et al., 1960→)

1



A non-trivial problem with several possible approaches (for students)

Comparing and discussion different ideas leads to new questions and problems

Point: students learn mathematical knowledge as rational solutions to problems

1

Simpler example (closer to university standard tasks)

Solve

$$3x^2 - 5x + 2 = 0 \qquad 1,\ 2/3$$
$$2x^2 - 5x + 3 = 0 \qquad 1,\ 3/2$$

What do you notice? Can you generalize it?

In fact, students can notice and prove, with various formulations and levels of generality:

For $q \neq 0$,

$$a_n q^n + \cdots a_1 q + a_0 = 0$$

if and only if

$$a_0 \left(\frac{1}{q}\right)^n + \cdots a_{n-1}\left(\frac{1}{q}\right) + a_n = 0$$

Essential phases in mathematical problem solving (Brousseau, 1997)

1

Action working with special cases (using "old" knowledge)
- Mathematical experiments

Formulation noticing and specifying "patterns"
- Matematical hypotheses
- Generalisation

Validation are the hypetheses true?
- Matematical reasoning, proof
- Counterexamples

Important guideline for task design: does working with the task enable students to engage in genuine mathematical activity?

And, even before that: what is the aim of students solving the task? What is the "target knowledge" ?

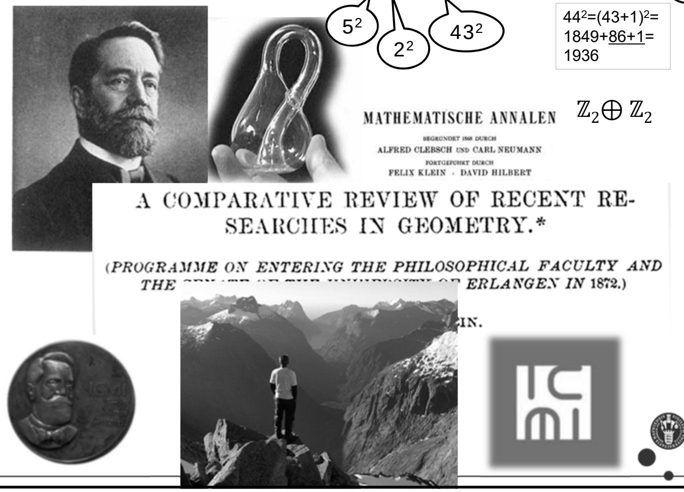
Didactic variables: vary the task but keep target fixed

Christian Felix Klein (25.04.1849-22.06.1925)

2

5^2 2^2 43^2

$44^2=(43+1)^2=$
$1849+\underline{86+1}=$
1936



$\mathbb{Z}_2 \oplus \mathbb{Z}_2$

Klein's Plan A and Plan B

2

In the history of mathematics, as well as in its teaching, we may identify two possible "plans":

***Plan A** is based upon a more particularistic conception of science, which divides the total field into a series of mutually separated parts and attempts to develop each part for itself, with a minimum of resources and with all possible avoidance of borrowing from neighbouring fields (Klein 1908, p. 78).*

While:
*… the supporter of **Plan B** lays the chief stress upon the organic combination of the partial fields, and upon the stimulation which these exert one upon another. He prefers, therefore, the methods which open for him an understanding of several fields under a uniform point of view. His ideal is the comprehension of the sum total of mathematical science as a great connected whole (ibid., p. 78).*

Modern university education overwhelmingly follows "plan A"

2

Effects of "mass university education" (Verret, 1975):
- **programmation** to make something *teachable,* organisation into *units* following a logic of economy for teachers and learners
  - Flexibility: units must be as *independent* as possible
  - Learnability: units tend to become *smaller and smaller*
- **desynchretisation**: contents which belong(ed) together gets separated
- **depersonalisation** (knowledge must be formulated independently of discovery context, e.g. timebound problems)

Paradigm of visiting monuments (Chevallard 2006):
In this fashion, [mathematical] praxeologies are soon turned into monuments, that is, things notable or great, fine or distinguished, but which, paradoxically, are effective in helping us to forget what they stand for – what exactly was the thing "monumentalised".

The need to recover the "big" questions – also outside mathematics

2

The teaching of mathematics is an old teaching, which has trouble getting renewed. What is it suffering from? **Basically from the escape, the exhaustion of making sense**.

Taught objects are condensed in **answers to questions that we have lost**. We need to recover these questions: Why are we interested in triangles? Why do we need to simplify fractions, or to rewrite a numerical expression in a canonical form? Why are we interested in the properties of figures? **There are so many questions that have lost their answers in a school culture turned into a lifeless 'museography'**.

Chevallard, Y. (2006). *Étudier et apprendre les mathématiques: vers un renouveau*

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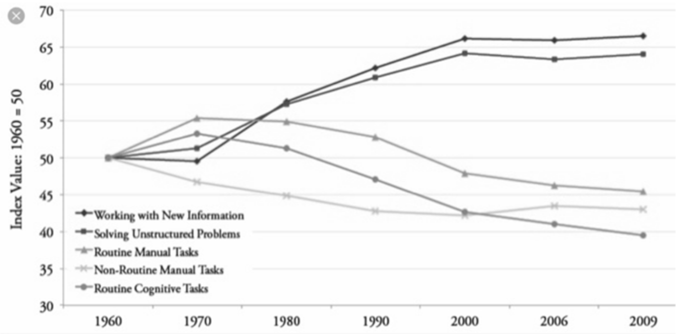
General mathematical needs of the future



www.thirdway.org
Dancing with Robots: Human Skills for Computerized Work - Third Way
Figure Index of Changing Work Tasks in the Economy 1960

2

Specific mathematical needs of students

What are the specific “mathematical needs” of your students?’
Largely unexplored in most cases (+ lack of methodology):

- What kind of *mathematical practice and theory* (MPT) is needed in professions students will pursue? With what level of autonomy? What role of technology ?
- How does MPT appear in the rest of students’ programme? In particular, what kinds of “big questions” call for MPT ?
- What situations (time, relationship to “main” courses) are optimal for students to learn the MPT needed ?

Didactics of Mathematics (also at university level) has traditionally been concerned mainly with “micro-level” questions, related to the teaching of given contents.

The determination of the contents and other “macro-level” organisational questions for university mathematics are only recently getting investigated.

Didactics of University Mathematics – an invitation

2

CERME 11

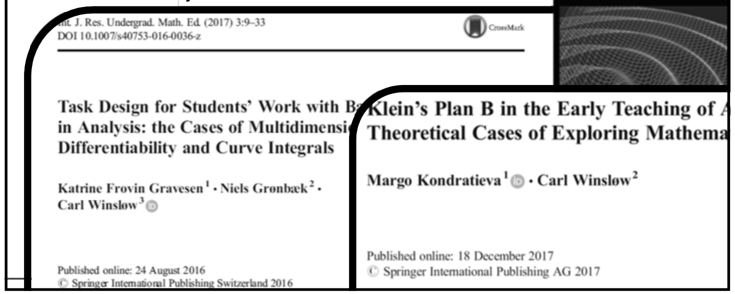
Utrecht, NL
Feb. 6-10, 2019
TWG 14 (of 25):
UME

INDRUM

Bizerte, Tunisia
March 27-29 2020
ETC conference on
UME

Also: UME groups at ICME and ICM
Annual RUME conferences in USA

And from 2016, a specialized journal : IJRUME



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Task Design for Students’ Work with B…
in Analysis: the Cases of Multidimensi…
Differentiability and Curve Integrals

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Klein’s Plan B in the Early Teaching of A…
Theoretical Cases of Exploring Mathema…

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